

## TRANSFORMATION OF PRE-SERVICE MATHEMATICS STUDENT'S ALGEBRA AND CALCULUS THINKING IN SOLVING DIFFERENTIAL EQUATION PROBLEMS

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### Abstract:

This study aims to analyze the transformation of algebraic thinking and calculus of preservice the mathematics students based on SOLO taxonomy in solving differential equations problems. The research subjects were 86 students in the mathematics education study program. Subject selection uses purposive sampling (students who take courses in differential equations). Data were collected using problem-solving tests and interviews which were then analyzed using the descriptive qualitative method with the following stages: (1) transcribing test and interview data, (2) coding segmentation, (3) analyzing student thinking transformations, and (4) concluding. The results showed that the transformation of algebraic and calculus thinking was used by students at each level of thinking to solve problems. The higher the level of thinking achieved, the better and the maximum transformation of algebraic and calculus thinking used by students. These results indicate that students need to be well supported and facilitated in problem-solving to achieve higher levels of thinking, such as the relational and extended abstract levels.

**Keywords:** Transformation Thinking, Algebraic Thinking, Calculus Thinking, SOLO Taxonomy

## TRANSFORMASI BERPIKIR ALJABAR DAN KALKULUS MAHASISWA MATEMATIKA DALAM MENYELESAIKAN MASALAH PERSAMAAN DIFERENSIAL

### Abstrak:

Penelitian ini bertujuan untuk menganalisis transformasi berpikir aljabar dan kalkulus mahasiswa berbasis taksonomi SOLO dalam menyelesaikan masalah persamaan diferensial. Subjek penelitian adalah 86 mahasiswa program studi pendidikan matematika. Pemilihan subjek menggunakan teknik purposive sampling yaitu mahasiswa yang menempuh mata kuliah persamaan diferensial. Pengumpulan data menggunakan tes pemecahan masalah dan wawancara yang kemudian dianalisis menggunakan metode kualitatif deskriptif dengan tahapan sebagai berikut: (1) menyalin data tes dan wawancara, (2) segmentasi pengkodean, (3) menganalisis transformasi berpikir siswa, dan (4) menarik kesimpulan. Hasil penelitian

menunjukkan bahwa transformasi berpikir aljabar dan kalkulus digunakan oleh siswa pada setiap tingkat berpikir untuk memecahkan masalah. Semakin tinggi tingkat berpikir yang dicapai, semakin baik dan maksimal transformasi berpikir aljabar dan kalkulus yang digunakan siswa. Hasil ini menunjukkan bahwa siswa perlu didukung dan difasilitasi dengan baik dalam pemecahan masalah untuk mencapai tingkat berpikir yang lebih tinggi, seperti tingkat relasional dan abstrak.

**Kata Kunci:** Transformasi Berpikir, Berpikir Aljabar, Berpikir Kalkulus, Taksonomi SOLO

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## INTRODUCTION

Studying differential equations (DEs) is a challenge, and the difficulty is caused by various factors such as knowledge of calculus (including multivariate calculus, real and complex analysis, vector analysis, optimization, and curve sketching), scientific knowledge, algebraic skills, competence in reading and applying theorems. Algebra and calculus play an important role in mathematics (McDowell, 2021; Rustika & Rohaeti, 2020; Sangwin, 2019). Calculus and algebra are also used as prerequisite materials that must be taken by students of the mathematics education study program if they want to take differential equations courses. Integral is used mainly as the inverse operation of the derivative and is widely used in DEs (Czocher, Tague, & Baker, 2013; Swastika, Nusantara, Subanji, & Irawati, 2019). The general finding regarding students' difficulties with the DEs solution is that students often do not conceptualize the solution as a function (Arslan, 2010; Raychaudhuri, 2014) but in the form of numbers.

The fundamental theorem of calculus relates certain integrals to the anti-differentiation process and serves to simplify the computation of definite integrals. However, this is not directly seen in DEs for two reasons (1) the differential equation itself is a collection of derivatives combined by various algebraic procedures that form conditions that are satisfied by the function, and (2) definite integrals are rarely found in the form

$\int_{\{a_1, \dots, a_n\}}^{\{b_1, \dots, b_n\}} f(x_1, \dots, x_n) dx_1 \dots dx_n$ . On the other hand, certain antiderivatives are selected based on initial conditions or boundary conditions known in DEs. The role of the fundamental theorem is further obscured by the fact that in DEs, integration is indirect and is generally often hidden in solving techniques using methods of indeterminate coefficients or variable separation (Czocher, Tague, & Baker, 2013).

There are many possible conditions in solving differential equation problems. First, not all DEs will have solutions so it is very useful to know the head of time if there is a solution or not. Second, DEs may have more than one solution. However, just because we know that a solution to a differential equation exists, doesn't mean we'll be able to find it. There are many ways to find solutions to DEs. Concluding these conditions, critical thinking is needed in solving DEs (Faradiba, Andriani, Alifiani, Walida, Daryono, Hasana, Angriani, Chamidah, Defitriani, & Qurohman, 2018).

Several problems regarding pre-service mathematics students' thinking processes in the differential equations (DEs) course were identified through observations during lectures and reflections after lectures. Students who take DEs courses already have provisions in previous courses (algebra and calculus) but often students are not able to use them when solving DEs problems. To solve DEs problems, it is necessary to have the ability to transform algebraic and calculus thinking into the context of DEs problems.

Thinking transformation is a mental activity in the thinking process. Increased ability in the thinking process will certainly increase the ability to solve problems. Research on thinking processes at the university level has been carried out by Lapp, Nyman, and Berry (2010) which shows that students find it more difficult to make connections between concepts such as eigenvalues and eigenvectors and from other conceptual parts such as bases and dimensions. Yantz (2013) also examined students in pre-calculus courses and concluded that students had not formed yet a relation between algebraic procedures and the fundamental properties of numbers.

Research on problem-solving has been carried out by several researchers and researchers themselves (Novitasari, Triutami, Wulandari, Rahman, & Alimuddin, 2020). Research that has been carried out by Socas and Hernandez (2013) shows that problem-solving is considered an inseparable part of mathematics and is explained in terms of problem solving, building relationships between concepts, operations, and processes implicit in

mathematical activities. Meanwhile, Carlson and Bloom (2005) produced a multidimensional problem-solving framework that has four stages, namely: orientation, planning, implementation, and review.

The problems that have been presented above give rise to research ideas with the theme of transformation of mathematical thinking. The mathematical thinking process is focused on the transformation of algebraic thinking and calculus in solving differential equations (DEs). The description of the transformation of thinking is explored with the SOLO Taxonomy framework (the Structure of the Observed Learning Outcome) which was first developed by Biggs and Collis in 1982 (Chick, 1998) and is grouped into five different hierarchical levels, level 0: pre-structural, level 1: uni-structural, level 2: multi-structural, level 3: relational, and level 4: extended abstract (Knapp, Adelman, Marder, McCoHum, Needels, Padilla, Shields, Turnbull, & Zucker, 1982).

Based on the background that has been described, this study aims to describe some of the qualitative characteristics of the transformation of algebraic and calculus thinking carried out by pre-service mathematics students in solving differential equations problems as reflected in the use of variables and calculations and solutions made by students. In addition, it also describes the extent to which students' responses show evidence of algebraic or arithmetical thinking and are related to calculus thinking. Differential equations were chosen because these subjects whose problem solving includes concepts of algebra and calculus, including polynomial/multi-term equations, derivatives, and integrals.

## **METHODS**

This study is an exploratory descriptive study with a descriptive quantitative approach (Johnson & Larry, 2004) which aims to analyze pre-service mathematics students' thinking processes, especially the transformation of students' algebraic and calculus thinking in solving differential equations problems. The research subjects consisted of 86 students of the Mathematics Education Study Program who took the differential equations course. The research instrument consisted of the researcher himself as the main researcher and a differential problem-solving test consisting of two questions to find out more about students' thinking processes. Students' thinking processes were analyzed based on the SOLO taxonomic thinking level which consists of 5 levels, namely pre-structural level (L0), uni-structural level (L1), multi-structural level (L2), relational level (L3), and extended

abstract level (L4). Several students representing each level of thinking in the SOLO taxonomy were interviewed to be able to find out more about the thinking transformations carried out by students.

Table 1. SOLO Taxonomy's Level

SOLO Taxonomy's Level	Description
Prestructural	Students' understanding of the problem is very limited that is not even interconnected, so they do not form a unified concept at all and do not have any meaning.
Uni-structural	Students can only respond to the questions that have been given but can not understand the responses they give so they can not get the correct answers.
Multi-structural	Students have the ability to respond the problems with several strategies of their own. Many connections they can make, but the connections are not right so still do not get the right answer.
Relational	Students can break a unit into several parts and determine how the parts are related to several models and can explain the equations of the model but they do not discover new principles and even have the wrong concept and students cannot apply the statement to the other cases.
Extended Abstract	Students can use all the information provided to solve the problems, students connect between information to get the right answer and students discover new principles and can prove the truth.

(Claudia, Kusmayadi, & Fitriana, 2020b, 2020a; Putri, Mardiyana, & Saputro, 2017)

The data that have been obtained were analyzed quantitatively using descriptive statistics and qualitative analysis by describing and providing an overview of the transformation process of algebraic thinking and calculus that is used by students in solving differential equation problems. The qualitative data analysis technique follows the concept given by Miles and Huberman (1994) with the stages: (1) transcribing the test and interview data, (2) coding segmentation, (3) analyzing student thinking processes in solving differential equation problems, and (4) conclusion.

## RESULTS AND DISCUSSION

The results of this study show that students' thinking skills are at the relational level (L3) and are able to reach the highest level of thinking in the SOLO taxonomy, namely the Extended Abstract Level as much as 68.60%. In questions 1 and 2, students' thinking skills are spread from Pre-Structural Level, Uni-Structural Level (L1), Multi-Structural Level (L2), Relational Level (L3) to Extended Abstract Level where students' thinking skills are dominant at Relational Level (L3) is 68.60% for question 1 and Multi-Structural Level (L2) for question 2 is 47.67%.

Problem 1 is "Determine the solution to the differential equation  $\frac{dy}{dx} + \frac{2}{3}x^2y = \frac{x^2}{3}$ " and problem 2 is "Determine the general solution of the following non-homogeneous differential equation  $y'' - 2y' + y = \frac{e^x}{x^2}$  using the parameter variation method". Table 2 below shows the description of students' thinking processes in solving problems at each level of thinking based on the result of student answer analysis and interview.

Table 2. Student Thinking Process Based on SOLO Taxonomy in Solving Differential Equation Problems

SOLO Taxonomy's Level	Descriptions	
	Item 1 (Q1)	Item 2 (Q2)
Pre-structural	<ul style="list-style-type: none"> <li>• Can understand the problem but at the stage of devising a plan, the concepts and information processing processes carried out are not relevant to the given problem.</li> <li>• The planned and chosen completion steps do not lead to the desired solution. Counting skills, algebraic and calculus thinking are used to solve problems but are not yet precise.</li> <li>• There are errors in the</li> </ul>	<ul style="list-style-type: none"> <li>• Can understand the problem but have not been able to use the information properly in devising a plan and implementing the solution.</li> <li>• The solution has no meaning or does not provide an answer to the question.</li> <li>• Unable to solve problems properly due to lack of skills. The algebraic thinking skills that were carried out were still not right and the concepts related to the completion of DEs using the parameter variation method were not well understood by the</li> </ul>

SOLO Taxonomy's Level	Descriptions	
	Item 1 (Q1)	Item 2 (Q2)
Uni- structural	<p>calculation process (the calculus skills used are still incorrect).</p> <ul style="list-style-type: none"> <li>• Did not re-check the answer.</li> </ul>	<p>subject. This can be seen from the solution completion process, most of which are still incorrect and have not been completed.</p> <ul style="list-style-type: none"> <li>• Did not re-check the answer.</li> </ul>
	<ul style="list-style-type: none"> <li>• Can understand the problem and be able to process the information on the problem appropriately.</li> <li>• In devising a plan, some subjects tried to use the variable separation method to solve the problem but when carrying out the solution, they failed to break down the two given equations to contain two equations with each corresponding variable. This causes the final result they obtained to be incorrect.</li> <li>• There are some conceptual errors. Starting from errors in algebraic operations/calculations to the integration process (in this case the subject's calculus knowledge cannot be used properly) so that the final results obtained are wrong.</li> <li>• Did not re-check the answer.</li> </ul>	<ul style="list-style-type: none"> <li>• Can understand the problem and use the information on the problem appropriately to devise a plan.</li> <li>• In carrying out the plan, there is a relation between one concept and another but the concept is not widely understood so the process is still wrong. This can be seen from the answers of the subjects whose process was not clear and some subjects only immediately gave the final result without the process and even then it was still incorrect.</li> <li>• Only focus on what they are looking for without understanding the value or meaning of the process they are doing.</li> <li>• The results they found were not correct.</li> <li>• Did not re-check the answer.</li> </ul>
Multi- structural	<ul style="list-style-type: none"> <li>• Can understand the problem and use the</li> </ul>	<ul style="list-style-type: none"> <li>• Can understand the problem</li> <li>• Information on the problem is</li> </ul>

SOLO Taxonomy's Level	Descriptions	
	Item 1 (Q1)	Item 2 (Q2)
	<p>information on the problem appropriately to devise a plan.</p> <ul style="list-style-type: none"> <li>• Can choose the right strategy to solve the problem by using the integral factorization method.</li> <li>• In carrying out the plan, their steps are structured / procedural, they are only wrong in the integration process (their calculus thinking skills are not right) so that the next process will also make the wrong results.</li> <li>• The conclusion written is also wrong. This is partly because the subject did not re-check the written answer.</li> </ul>	<p>used correctly in devising a plan where is the process of solving the problem using the Cramer method and the substitution elimination method after knowing the general solution of homogeneous Des.</p> <ul style="list-style-type: none"> <li>• The completion steps are carried out procedurally and systematically. The answer produces the desired or needed information and data to solve the problem but there is an error in the next calculation process. Counting and algebraic thinking skills have not been carried out optimally. The concept of using the Cramer Method in helping to determine special solutions to differential equations (DEs) is not well understood (the formulas and answers are written incorrectly).</li> <li>• Did not re-check the answer.</li> </ul>
Relational	<ul style="list-style-type: none"> <li>• Can understand the problem and use the information on the problem appropriately to devise a plan</li> <li>• Carrying out the plan by following the plan where the completion steps are carried out algorithmically and systematically correctly (counting skills, algebra,</li> </ul>	<ul style="list-style-type: none"> <li>• Can understand the problem and use the information on the problem appropriately to devise a plan.</li> <li>• Carrying out the plan by following the plan where the completion steps are carried out algorithmically and systematically correctly (counting skills, algebra, and calculus used are correct).</li> <li>• The final result is correct, but</li> </ul>



SOLO Taxonomy's Level	Descriptions	
	Item 1 (Q1)	Item 2 (Q2)
	<p>and calculus used are correct) only when determining the integral factor, the subject has not completed it.</p> <ul style="list-style-type: none"> <li>• The resulting answer has fulfilled what was asked in the question and correct.</li> <li>• Some subjects did not write down the conclusions of the work carried out.</li> </ul>	<p>the process of finding a solution is not written in a systematic and inaccurate manner.</p> <ul style="list-style-type: none"> <li>• The resulting answer has fulfilled what was asked in the question and correct.</li> <li>• Some subjects did not write down the conclusions of the work carried out.</li> </ul>
Extended Abstract	<ul style="list-style-type: none"> <li>• Can understand the problem and use the information on the problem appropriately to devise a plan.</li> <li>• Devising a plan by choosing the right solution strategy.</li> <li>• Can generalize and relate the skills of counting, thinking algebra, and calculus correctly to solve the problem. The solution steps their use are carried out algorithmically and systematically.</li> <li>• The answer given has fulfilled what was asked by the question and is correct.</li> <li>• Re-check answers and write conclusions that are relevant to the problem.</li> </ul>	<ul style="list-style-type: none"> <li>• Can understand the problem and use the information on the problem appropriately to devise a plan.</li> <li>• The completion steps are carried out algorithmically and systematically correctly.</li> <li>• Can understand existing concepts to solve problems (determine the general solution of homogeneous DEs in the form of iterative complex roots of characteristic equations and skillful algebraic calculations performed appropriately). The process of integration and algebraic calculations are done correctly.</li> <li>• The answer given has fulfilled what was asked by the question and is correct.</li> <li>• Re-check answers and write conclusions that are relevant to the problem.</li> </ul>

$\frac{dy}{dx} + \frac{2}{3}x^2y = \frac{x^2}{3} \Rightarrow \frac{2}{3}x^2y = \frac{x^2}{3} \cdot \frac{dx}{dy}$   
 $\frac{2}{3}x^2y dy = \frac{x^2}{3} dx \Rightarrow \left(\frac{2}{3}x^2y\right) dy - \left(\frac{x^2}{3}\right) dx = 0$   
 • Cek kondisi keakraban:  
 $M = \frac{x^2}{3} \Rightarrow \frac{\partial M}{\partial y} = \frac{x^2}{3}$   
 $N = \frac{2}{3}x^2y \Rightarrow \frac{\partial N}{\partial x} = \frac{4}{3}xy$   
 Bukan PD Eksak  
 Misalkan solusi umumnya  $u(x,y) = C$   
 Karena  $\frac{\partial u}{\partial x} = M$ , maka  $u = \int M dx$   
 $= -\frac{1}{6}x^3 + k(y)$   
 Sehingga diperoleh  $k(y) = C$ .  
 Jadi, solusi umumnya adalah  $-\frac{1}{6}x^3 + C = 0$

The concept and processing of information are not relevant to the problem

The result of the integration is incorrect

Figure 1. Example of The Subject's Answer Problem 1 on the Pre-Structural Level

Penyelesaian:  
 •) solusi umum PD homogen  
 Persamaan differensial homogennya  
 $y'' - 2y' - 9 = 0$   
 Persamaan bantu:  $r^2 - 2r - 9 = 0$   
 $r_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$   
 $= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-9)}}{2}$   
 $= \frac{2 \pm \sqrt{4 + 36}}{2}$   
 $= \frac{2 \pm \sqrt{40}}{2}$   
 $= 1 \pm \sqrt{10}$   
 Dimisalkan solusi khususnya,  $y_p = V_1 e^{r_1 x} + V_2 e^{r_2 x}$  dgn syarat  
 $V_1 e^{r_1 x} + V_2 e^{r_2 x} = 0$   
 $V_1 e^{-r_1 x} - V_2 e^{-r_2 x} = \frac{e^x}{x^2}$   
 $r_{1,2} = \frac{2 \pm \sqrt{40}}{2}$   
 $r_1 = 1 - \sqrt{10} \quad r_2 = 1 + \sqrt{10}$

Figure 2. Example of The Subject's Answer Problem Problem 2 on The Pre-Structural Level

Figure 1 shows that the subject already knew that the DEs in the problem are not exact DEs but the subject still uses the exact DEs solution to solve the problem. What they did was incorrect which resulted in the wrong result. Even the integration is still incorrect. Also figure 2 shows that algebraic skills in determining the roots of an equation are incorrect.

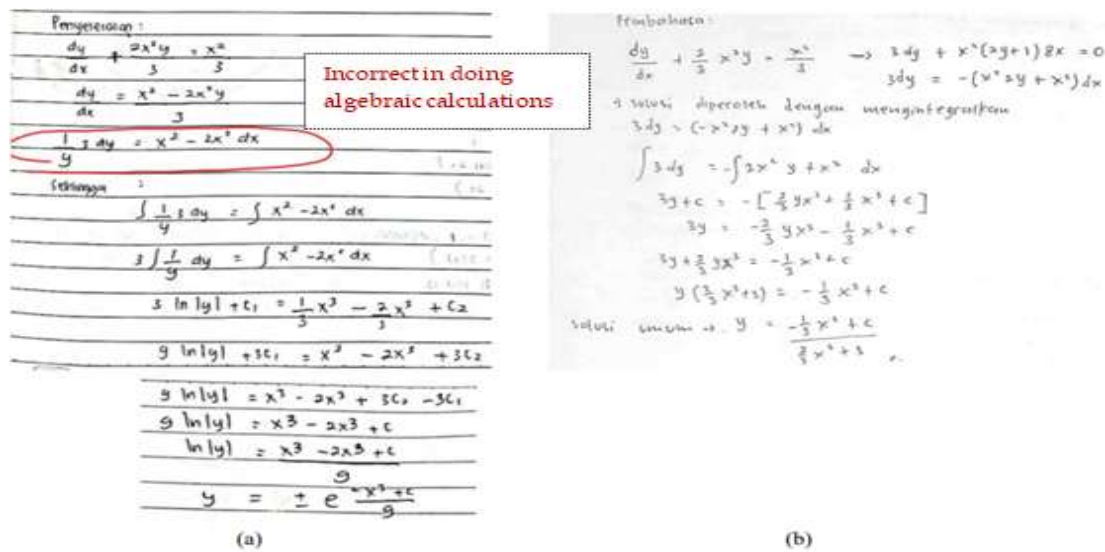


Figure 3. Example of Subject's Answer Problem 1 on Uni-Structural Level

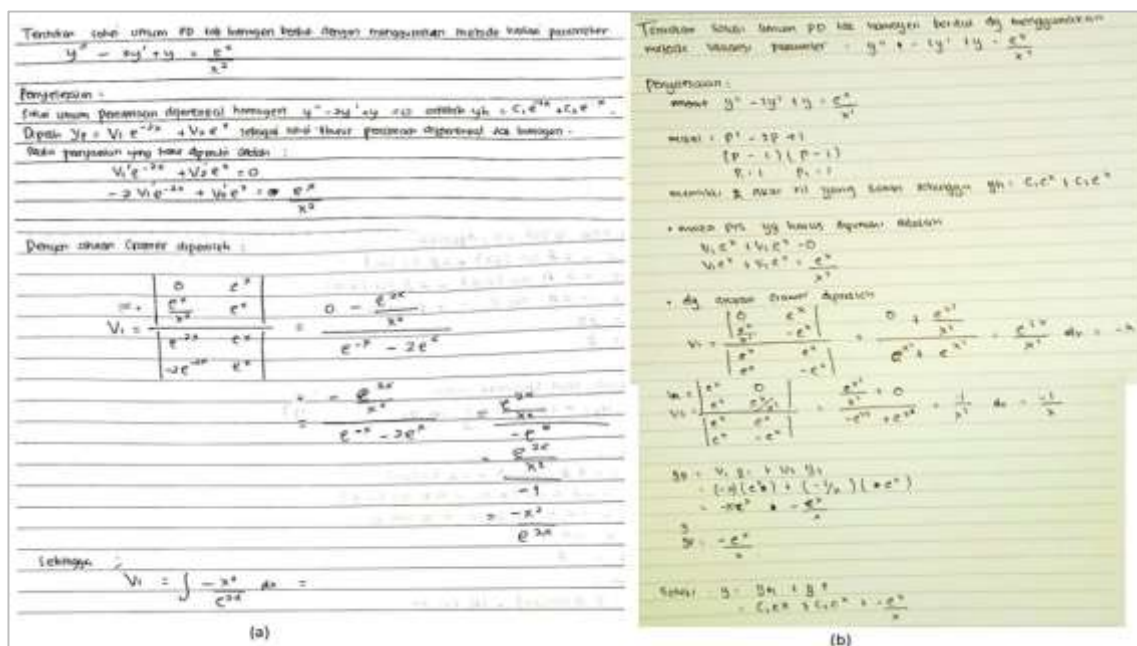


Figure 4. Example of Subject's Answer Problem 2 on Uni-Structural Level

Figures 3 and 4 show that in carrying out the plan there is a relation between one concept and another that is carried out by the subject but there are errors in the algebraic thinking skills carried out. The subject is also known to have not understood the meaning of the completion process he did. They only focus on work without understanding the meaning of what they are doing.

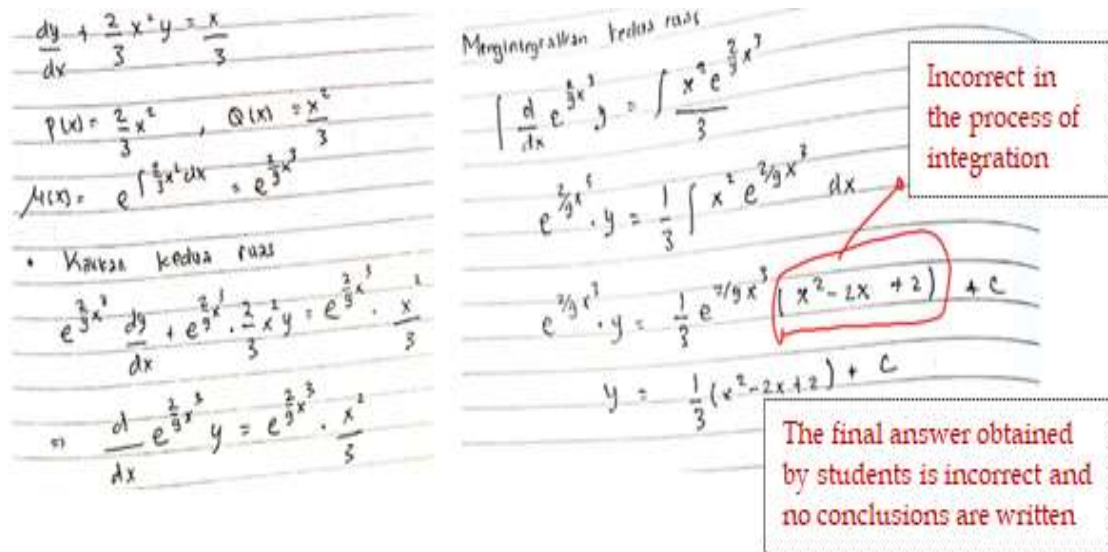


Figure 5. Example of The Subject's Answer Problem 1 on The Multi-Structural Level

Based on figure 5, it can be seen that there is a transformation of calculus and algebraic thinking in solving the problem but still wrong which causes the final answer also to be wrong.

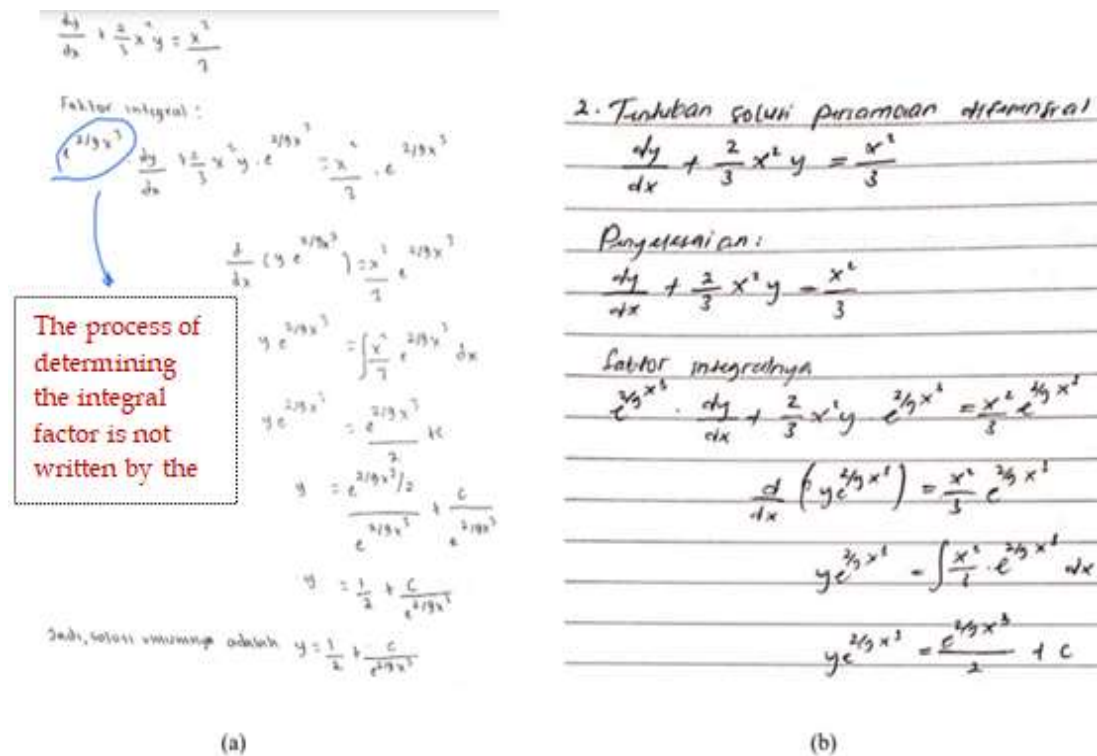


Figure 6. Examples of The Subject's Answers Problem 1

Jawab:

PD Homogen :  $y'' - 2y' + y = 0$   
 Pers. bantu :  $r^2 - 2r + 1 = 0$   
 $r_1 = 1$

Solusi umum  $y_h = C_1 e^x + C_2 e^{-x}$

PD Non homogen :

$V_1' e^x + V_2' e^{-x} = 0$   
 $V_1' e^x + V_2' e^x + x e^x = \frac{e^x}{x^2}$

Dengan metode cramer diperoleh:

$$V_1' = \frac{\begin{vmatrix} 0 & e^x \\ e^x & e^x + x e^x \end{vmatrix}}{\begin{vmatrix} e^x & e^x \\ e^x & e^x \end{vmatrix}} = \frac{-\frac{e^{2x}}{x}}{e^{2x}} = -\frac{1}{x} = \int -\frac{1}{x} dx = -\ln(|x|)$$

$V_2$

Dikalikan solusi khusus PD tak homogen adalah

$y_p = -\ln(|x|) e^x - \frac{1}{x} (x e^x)$  atau

$y_p = -\ln(|x|) e^x - e^x$

Jawab solusi umum PD tak homogen adalah

$y = C_1 e^x + C_2 e^{-x} - \ln(|x|) e^x - e^x$

Figure 7. Examples of The Subject's Answers Problem 1 on Relational Level

Figure 6 and figure 7 show that the final result written is correct, it's just that the process written in finding the solution is still not systematic and there are still steps that are not written down.

<p>Tentukan solusi persamaan diferensial</p> $\frac{dy}{dx} + \frac{2}{3} x^2 y = \frac{x^2}{3}$ <p>Penglesaian:</p> <p>Faktor integrasinya <math>e^{\int \frac{2}{3} x^2 dx} = e^{\frac{2}{9} x^3}</math></p> <p>Kedua ruas dikalikan faktor integrasinya</p> $e^{\frac{2}{9} x^3} \frac{dy}{dx} + \frac{2}{3} x^2 y e^{\frac{2}{9} x^3} = \frac{x^2}{3} e^{\frac{2}{9} x^3}$ $\frac{d}{dx} (y e^{\frac{2}{9} x^3}) = \frac{x^2}{3} e^{\frac{2}{9} x^3}$ $y e^{\frac{2}{9} x^3} = \int \frac{x^2}{3} e^{\frac{2}{9} x^3} dx$ $y e^{\frac{2}{9} x^3} = \frac{1}{2} e^{\frac{2}{9} x^3} + C$ $y = \frac{1}{2} + \frac{C}{e^{\frac{2}{9} x^3}}$ <p>Solusi umumnya adalah <math>y = \frac{1}{2} + \frac{C}{e^{\frac{2}{9} x^3}}</math></p> <p>(a)</p>	<p>Tentukan solusi persamaan diferensial</p> $\frac{dy}{dx} + \frac{2}{3} x^2 y = \frac{x^2}{3}$ <p>Penglesaian:</p> $\frac{dy}{dx} + \frac{2}{3} x^2 y = \frac{x^2}{3}$ <p>faktor integral <math>e^{\int \frac{2}{3} x^2 dx} = e^{\frac{2}{9} x^3}</math></p> $e^{\frac{2}{9} x^3} \frac{dy}{dx} + \frac{2}{3} x^2 y e^{\frac{2}{9} x^3} = \frac{x^2}{3} e^{\frac{2}{9} x^3}$ $\frac{d}{dx} (y e^{\frac{2}{9} x^3}) = \frac{x^2}{3} e^{\frac{2}{9} x^3}$ $y e^{\frac{2}{9} x^3} = \int \frac{x^2}{3} e^{\frac{2}{9} x^3} dx$ $y e^{\frac{2}{9} x^3} = \frac{1}{2} e^{\frac{2}{9} x^3} + C$ $y = \frac{1}{2} + \frac{C}{e^{\frac{2}{9} x^3}}$ <p>Jadi, solusi PDnya adalah <math>y = \frac{1}{2} + \frac{C}{e^{\frac{2}{9} x^3}}</math></p> <p>(b)</p>
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Figure 8. Example of The Subject's Answer Problem 1

<p>Penyelesaian:</p> <p>Solusi umum persamaan diferensial homogen <math>y'' - 2y' + y = 0</math> adalah <math>y_h = C_1 e^x + C_2 x e^x</math>.</p> <p>Ditrich <math>y_p = v_1 e^x + v_2 x e^x</math> dicari untuk mencari persamaan diferensial tak homogen.</p> <p>Menentukan SPL:</p> $v_1' y_1 + v_2' y_2 = 0 \rightarrow v_1' e^x + v_2' x e^x = 0$ $v_1' u_1 + v_2' u_2 = g(x) \quad v_1' e^x + v_2' (e^x + x e^x) = \frac{e^x}{x^2}$	<p>* Selesai mendapat <math>v_1</math> dan <math>v_2</math> kemudian masukkan <math>v_1</math> dan <math>v_2</math></p> $v_1 = \int v_1' dx = \int -\frac{1}{x} dx = -\int \frac{1}{x} dx = -\ln x  + c$ $v_2 = \int v_2' dx = \int \frac{1}{x^2} dx = \int x^{-2} dx = -x^{-1} + c = -\frac{1}{x} + c$
<p>Penyelesaian SPL</p> $v_1' e^x + v_2' x e^x = 0$ $v_1' e^x + v_2' (e^x + x e^x) = \frac{e^x}{x^2}$ $\frac{v_1' (e^x)}{e^x} = -\frac{e^x}{x^2}$ $v_1' = -\frac{e^x}{x^2}$ $v_1 = \frac{1}{x}$	<p>* Kemudian substitusikan <math>v_1</math> dan <math>v_2</math> ke <math>y_p = v_1 y_1 + v_2 y_2</math></p> $y_p = v_1 e^x + v_2 x e^x$ $y_p = -\ln x  e^x + \left(-\frac{1}{x}\right) x e^x = -\ln x  e^x - e^x$
<p>Selesai:</p> $v_1' e^x + v_2' x e^x = 0$ $v_1' e^x + \left(\frac{1}{x^2}\right) x e^x = 0$ $v_1' e^x + \frac{1}{x} e^x = 0$	<p>* Selesai di dapatkan:</p> $y = y_h + y_p$ $= C_1 e^x + C_2 x e^x - \ln x  e^x - e^x$ $= (C_1 - 1) e^x + C_2 x e^x - \ln x  e^x$ $y = C_1 e^x + C_2 x e^x - e^x \ln x $ <p><math>C_1 - 1 =</math> misalkan konstanta jadi dapat ditulis <math>C_1</math></p>

Figure 9. Example of The Subject's Answer Problem 2 on The Extended Abstract Level

Based on the results of student answers and interviews conducted with several selected student representatives based on the SOLO taxonomy thinking level, students gave different responses to solve problems. Starting from students with the lowest response rate (Pre-Structural Level), where: (1) Students generally understand the problem but cannot process the information obtained properly, (2) Do misconceptions both in terms of concepts to devising a plan and thinking skills algebra and calculus that they use, (3) When carrying out the plan, the steps they use are generally meaningless, and (4) They tend not to give the results that are asked for in the questions. Many algebraic and calculus calculations were made wrong. The answer given is not enough, where the problem does not lead to the solution it should have. The method chosen has not yet led to the desired solution. These results are also supported by several previous studies that have been conducted (Chick, 1998; Claudia, Kusmayadi, & Fitriana, 2020b, 2020a; Lian & Yew, 2012; Upu & Bangatau, 2019) related to thinking criteria at the Pre-Structural Level. However, this study also found that the students' ability to think algebraic and calculus still made a lot of mistakes.

The next level is the Uni-Structural level where (1) Students are able to understand the given problem but do not use the information properly in solving problems. Although in devising a plan, students plan to use it they have difficulty. (2) When carrying out the plan, the subject only focuses on the solution that they will use without understanding the value or the meaning of the equation or the data they get; (4) The results they found were not quite correct. The subject only focuses on the problem. Their answers are not systematic and some subjects just give the final result without any process and even then it is still not correct. The transformation of algebraic and calculus thinking is not very visible because the process is not described. Do not understand the value of the existing data and do not understand the relationship between the data and others that have been calculated previously, so students' answers are inconsistent. (5) Students do not check the written answers. This result is also supported by several previous studies that have been conducted (Claudia, Kusmayadi, & Fitriana, 2020a; Lian & Yew, 2012; Upu & Bangatau, 2019) related to thinking criteria at the Uni-Structural Level. However, this study also found that the students' ability to think algebraic and calculus are not very visible.

The next level is the multi-structural level where: (1) The subject understands the problem; (2) The information in the questions is used correctly to devise a plan, (3) At the stage of carrying out the plan, the subject takes steps procedurally and systematically but has not been able to connect the information with the existing concepts. Their answer produces the desired or needed information and data to solve the problem but there are errors in the further calculation process (4) Counting skills, algebraic and calculus thinking have not been carried out optimally but are used quite well in solving problems. They made a mistake in the algebraic calculation process. (5) Not checking the written answers. This result is also supported by several previous studies that have been conducted (Afriyani, , Subanji, & Muksa 2018; Chick, 1998; Claudia, Kusmayadi, & Fitriana, 2020a; Lian & Yew, 2012; Saputra, Nurjanah, & Retnawati, 2019) related to thinking criteria at the Multi-Structural level.

The next level is the Relational level where: (1) Students can understand the given problem and can process the information appropriately, (2) The planned settlement strategy is appropriate, and (3) The completion steps are selected and carried out algorithmically and systematically. Counting skills, algebraic and calculus thinking skills used are correct (3) Students understand

the existing concepts, (4) The answers produced are fulfilled and correct, and (5) Unfortunately students have not been able to conclude these questions. Students can use all the information so that they can provide answers to problems and understand the meaning of the information and can understand the relationship between data. As a result, they can build a structure consistently, and can connect the existing concepts as a whole in a harmonious and meaningful way. (6) Checking the written answers. This result is also supported by several previous studies that have been conducted (Claudia, Kusmayadi, & Fitriana, 2020b, 2020a; Lian & Yew, 2012; Saputra, Nurjanah, & Retnawati, 2019) related to thinking criteria at the Relational level.

The last level is Extended Abstract where: (1) Students can understand the problem and the information on the problem is used properly in planning problem-solving. Students at this level can use 2 different methods in solving problems correctly, (2) The relationship between the concept and the application of information has been well structured, and (3) In carrying out the plan, the steps are used algorithmically and systematically. Counting skills, algebraic and calculus thinking skills are used well. In addition, there is a transformation of algebraic and calculus thinking used by the subject in solving the problems given correctly, 4) the answers they have given have fulfilled what was asked by the questions and were correct, and (5) Students were able to provide conclusions from the problem. This result is also supported by several previous studies that have been conducted (Claudia, Kusmayadi, & Fitriana, 2020b, 2020a; Putri, Mardiyana, & Saputro, 2017). This result is slightly different from the research conducted by Upu & Bangatau (2019) where at this level students cannot find the right answer, while in this study, the subjects were able to solve the problem correctly based on the information obtained by involving the transformation of algebraic and calculus thinking well.

## **CONCLUSION**

Based on the results of data analysis, it can be concluded that the transformation of algebraic and calculus thinking is used by students at each level of thinking to solve problems. The higher the level of thinking achieved, the better the transformation of algebraic and calculus thinking used by students. Thinking transformations carried out by students (1) Pre-Structural Level: students generally understand the given problem but cannot process the information obtained properly. Many of the algebra and calculus calculations



that students do are wrong. The answers given are not sufficient, where the problem does not lead to the desired solution, (2) Uni-Structural Level: students do not use the information on the problem properly. The transformation of algebraic and calculus thinking is not very visible because students do not describe the process. (3) Multi-Structural Level: students understand the problems and devise the plan well. Counting skills, algebraic thinking and calculus have not been carried out optimally so that they produce wrong answers but are used quite well in solving problems, (4) Relational Level: students can process information correctly. The selected completion steps are carried out algorithmically and systematically. Counting skills, algebraic thinking skills and calculus used is correct but cannot conclude these questions (5) Extended Abstract: students can understand concepts well. The completion steps are used algorithmically and systematically. Counting skills, algebraic thinking skills, and calculus are used very well and can provide conclusions from problems.

Based on research, educators especially lecturers need to train students more related to problem-solving, concepts needed in problem-solving, and algebraic and calculus thinking skills. Both from the Pre-Structural level to the highest level.

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